

Multi-User System for Earth Sensing Attitude Estimation Analysis Using Spectral Decomposition

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The Teledyne Multiple User System for Earth Sensing (“MUSES”) was developed for commercial earth imaging and instrument technology development on the International Space Station. MUSES simultaneously hosts up to four Earth sensing instruments and provides inertial pointing and tracking to orient instruments toward planned targets on the ground. Up to four instruments are robotically installed on MUSES and periodically interchanged as warranted. MUSES was delivered to the International Space Station in June 2017 with initial testing successfully completed and subsequently documented in an initial requirements verification report. Assessing the performance of the MUSES attitude estimation system is challenging due to lack of an independent attitude estimate of the platform attitude. A novel approach is introduced herein where the ISS attitude estimate, derived from ISS GPS sensors, is employed as an independent measure of MUSES platform attitude. Spectral decomposition methods are used to account for the contributions of ISS deformations to variance in disparities between the ISS and MUSES attitudes and thus enable a less conservative estimate of the variance in MUSES attitude error. Implementing the spectral decomposition results in a best estimate upper bound on MUSES attitude errors of 34.5 and 27.7 arc seconds, respectively in along-track and cross-track axes.

I. Introduction

The Teledyne MUSES system implements a two-axis (outer/inner gimbal configuration) inertial feedback pointing control system for precise pointing of instruments with respect to Earth surface targets. As shown in Fig. 1, the MUSES Pointing Control System includes gimbal actuators to move the platform in two axes; sensors to measure the motion and orientation of the platform; input data representing the ISS state and targeting commands from the ground; and pointing control algorithms to execute the orbit determination, state estimation, pointing guidance, and pointing control functions. Inertial rate measurements in three axes are provided by a platform-mounted Honeywell Miniature Inertial Measurement Unit (MIMU), sampled at 50 Hz, and during dark periods of star acquisition, inertial attitude estimates are obtained from a Sodern SED26 autonomous star tracker, sampled at 10 Hz. Control torques are applied to the platform with MOOG stepper motor gimbals on the inner and outer axes, resulting in platform line of sight motion in the along and cross-track directions.

Properly relating the sensors signals and gimbal articulations to the guidance commands for pointing the platform involves coordinate transformations between multiple coordinate frames. The MUSES Platform Coordinate Frame shown in Fig. 2 is fixed to the MUSES platform and rotates with the platform and instruments. The MUSES Pointing Control System converts mode-specific targeting commands into track-relative angle and rate commands that are referenced to the Local Vertical Local Horizontal (LVLH) frame as shown in Fig. 3. The LVLH frame is an ISS body-fixed frame with the positive z-axis pointing toward Earth’s center, the positive y-axis opposite of the orbit angular momentum vector, and the positive x-axis completing the right hand coordinate system. Guidance commands in an earth centered reference frame are converted to track angle commands in the LVLH frame as a function of the time varying ISS position, the location and orientation of the MUSES Mount Frame (not shown) relative to the ISS body frame, and the orientation of the MUSES Platform Frame (e.g. gimbal angles) relative to the MUSES Mount Frame. Hence correctly estimating the inertial orientation of the MUSES platform is essential for accurate pointing and geolocation of images acquired from hosted instruments.

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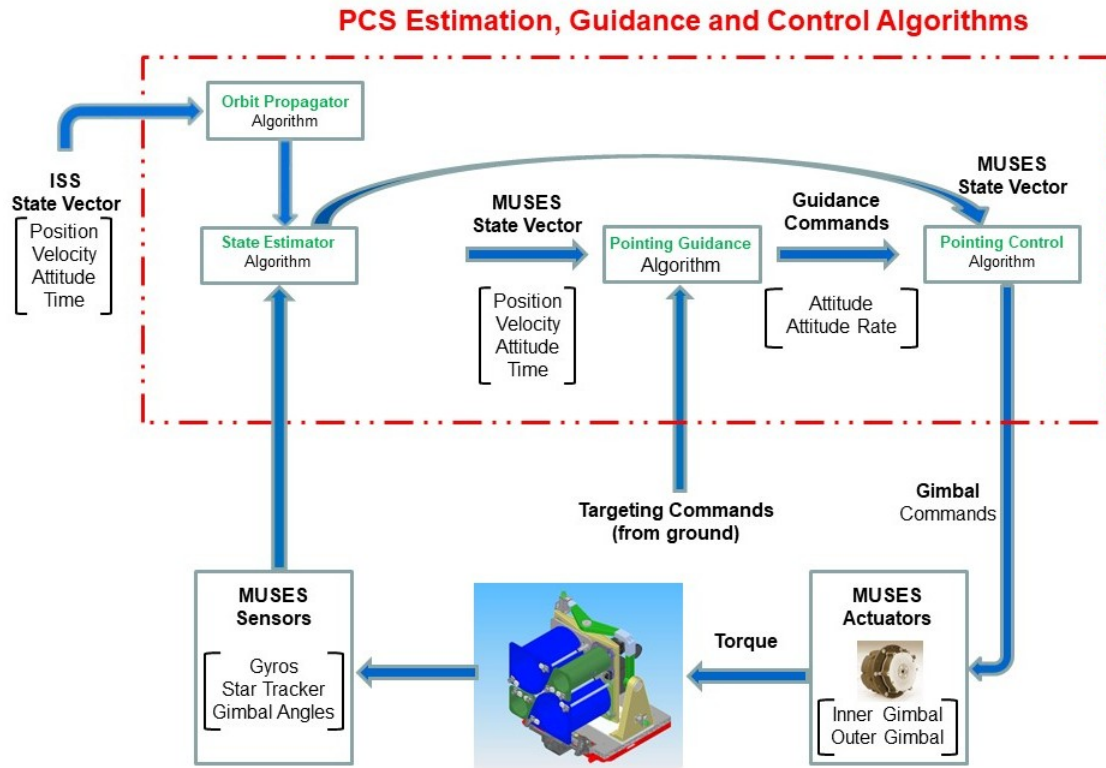


Fig. 1 MUSES Pointing Control System Architecture.

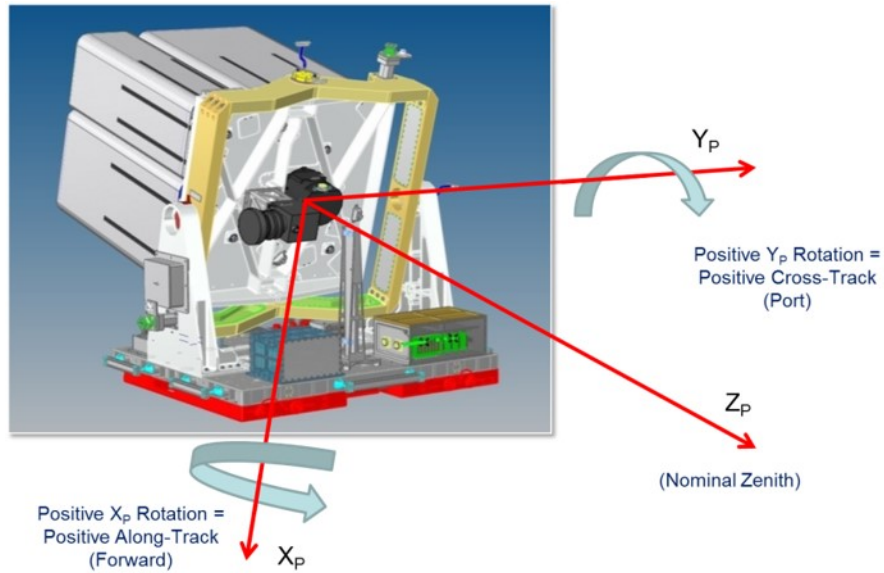


Fig. 2 MUSES Platform Frame (Section 2.2.7 of Ref. 1).

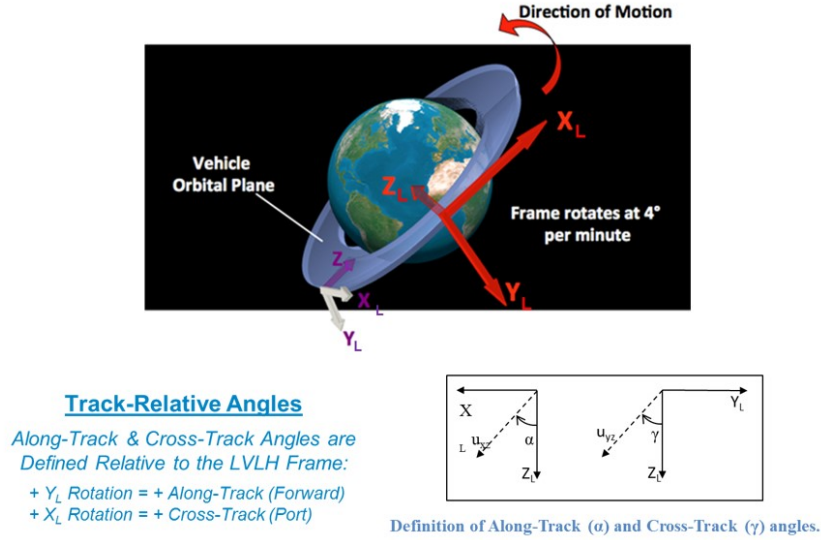


Fig. 3 LVLH Frame and Definition of Pointing Command Angles (from Section 2.3.5 of Ref. 1).

II. MUSES Attitude Estimation

The MUSES Pointing Control System (PCS) generates an estimate of the platform orientation in inertial space using an Extended Kalman Filter (EKF) to optimally blend the SED26 star tracker attitude measurements (during star acquisition periods) and MIMU incremental angles. The MUSES Platform inertial orientation is projected into the LVLH frame to obtain the track-relative along-track and cross-track angle estimates. The current flight version of the state estimation algorithm differs in significant ways from the initial design documented in Ref. 1 and hence this section will briefly describe the key difference that pertains to attitude estimation performance and analysis. A more complete description of the flight algorithms can be found in Ref. 1 and a development of the revisions to the EKF can be found in Ref. 2.

The original MUSES EKF documented in Ref. 1 includes filter states for star tracker measurement bias and correctly notes that the star tracker biases are unobservable due to the lack of an additional sensor providing absolute, unbiased attitude measurements. The inclusion of unobservable star tracker bias filter states is said therein to enable accounting for the uncertainty in the star tracker bias and more accurately estimate the attitude error. However, Ref. 2 conclusively shows that the original formulation of the MUSES EKF is incorrect in accounting for the effect of SED26 star tracker bias on the MUSES attitude estimate error. Ref. 2 develops the correct mathematical formulation of the EKF, demonstrates the resulting rank deficiency of the filter observability matrix and presents test cases that show unbounded attitude errors resulting from modest values of star tracker bias.

To remedy this deficiency and render the star tracker biases observable in the filter states requires an independent and unbiased means to observe the star tracker offsets in the image focal plane. Typical spacecraft attitude determination systems employ multiple star trackers for this purpose, but with only a single star tracker, MUSES must rely on an unbiased estimate of the ISS attitude derived from the ISS GNC (Guidance, Navigation and Control) system to infer the inertial orientation of the MUSES platform inertial orientation as described below.

ISS state estimates are obtained by using GPS data from multiple GPS sensors to produce rigid body attitude measurements of the ISS in addition to traditional position, velocity, and time determination. When combined with ISS rate gyro data, the ISS attitude estimates meet the ISS GNC knowledge requirement of 0.5 deg/axis, 3 sigma and typically err by less than 0.25 deg/axis, 3 sigma. However, the attitude knowledge specification for external (non-articulated) locations on the ISS is 3 deg/axis, 3 sigma, which necessitates the use of dedicated inertial sensors on the MUSES platform to more accurately sense the inertial orientation of the MUSES platform. (Specifications are from the briefing “ISS GNC Performance Requirements & Analytic Predictions”, ISS Guidance, Navigation, and Control Team, September 20th, 2010, Yared Mesfin).

Initially the concern had been that due to the significantly larger attitude error in the ISS attitude estimate (1 degree standard deviation) as compared to the star tracker error (arcseconds), the ISS attitude estimate would not be sufficiently accurate to estimate the star tracker biases. However, over sufficiently long periods of time (many hours based on

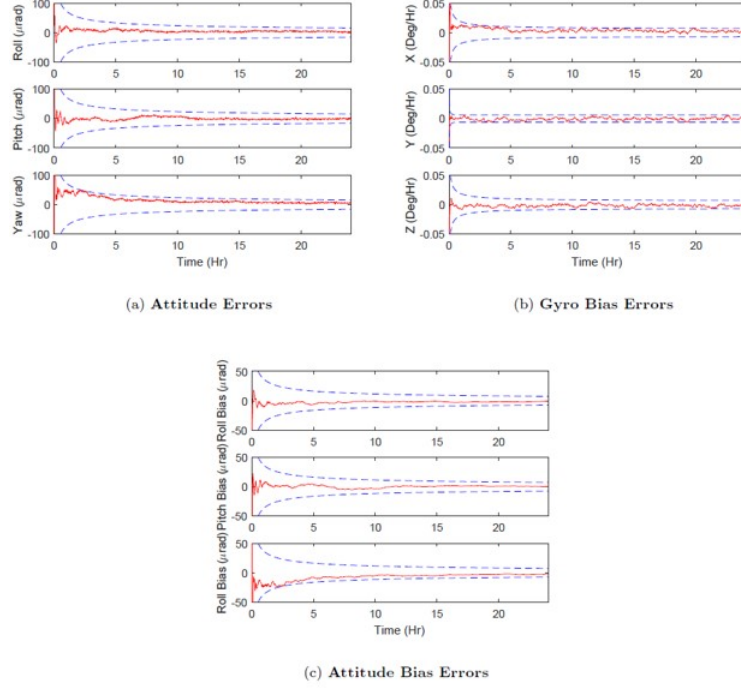


Fig. 4 EKF Attitude Estimation Performance with ISS attitude state blending (originally Fig. 5 in Ref. 2)

analysis in Ref. 2), the unbiased ISS attitude estimate reduces the attitude error in the MUSES EKF so that the star tracker bias is accurately estimated and significantly increased accuracy of the MUSES attitude estimate results. Figure 4 is taken from Ref. 2 and shows the resulting attitude, gyro biases, and attitude biases from a simulation of the MUSES EKF over 24 hours.

It is worth emphasizing that while the ISS state estimate does not meet the pointing knowledge accuracy requirements of the MUSES PCS, its use as a measurement in the MUSES EKF renders the MUSES star tracker bias observable and thus enables the MUSES EKF to accurately estimate the inertial pointing attitude of the MUSES platform.

Not only is the ISS state vector necessary to accommodate star tracker bias in the MUSES attitude estimate, the ISS state vector is essential for an independent estimate. The following section develops the mathematical basis for estimating MUSES attitude error by assessing the disparity between the MUSES attitude estimate and the ISS attitude estimate

III. Analysis of MUSES Attitude Estimation Performance

Assessing the performance of the MUSES PCS state estimation requires a reference inertial attitude measurement for comparison with the MUSES attitude estimate. Imagery from MUSES hosted instruments may eventually provide independent inertial reference data points through image geolocation, but for the purposes of statistically meaningful and operational performance assessment, the only independent inertial reference measurement is the ISS state vector.

Both time domain and frequency domain measures are employed to assess the accuracy of the MUSES attitude estimate. Estimates of MUSES attitude are compared to ISS attitude estimates to form the Track-Angle Disparity in each axis, and frequency domain methods are employed to estimate and remove the contribution of orbit-driven ISS deformations from the relative alignments of the ISS and MUSES fixed reference frames.

The resulting Residual Attitude Error is the best estimate of MUSES attitude estimation performance than can be obtained from flight data and provides a useful tool for routinely monitoring MUSES nominal operations.

It should be noted that while the ISS state is an optimal, minimal error estimate, residual ISS error is blended with MUSES EKF estimation error in this estimate of MUSES attitude estimation error. Nonetheless the Residual Attitude Error is the best estimate of MUSES attitude estimation performance absent other inertial attitude sensors.

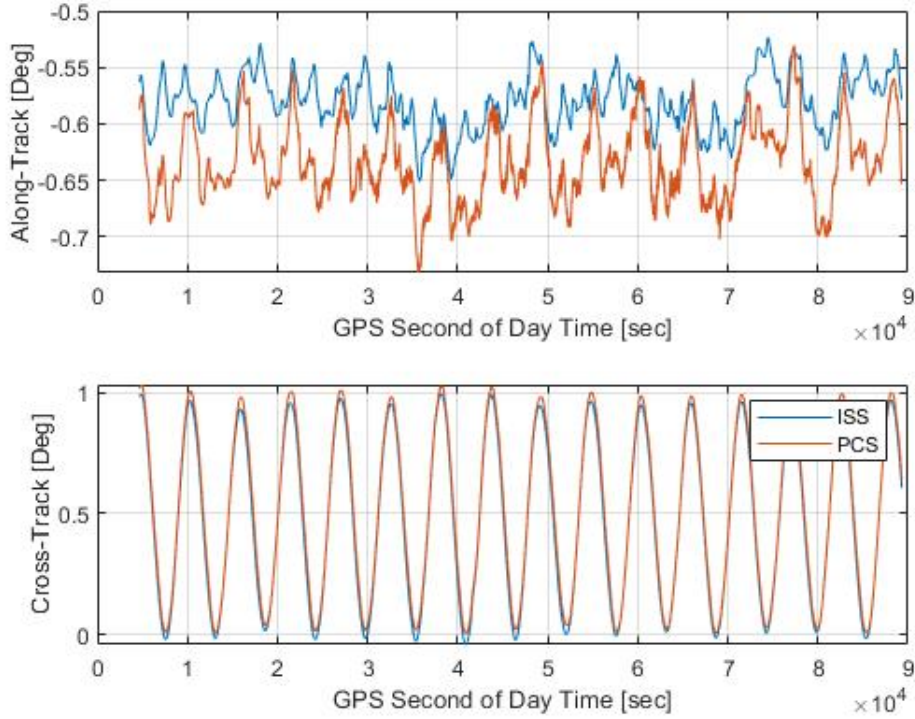


Fig. 5 ISS and PCS Track Angles, January 5, 2018

A. Track Angle Disparities

Accuracy of the MUSES PCS attitude estimate can be inferred by comparing the MUSES PCS attitude estimate and the ISS attitude estimate in a common coordinate frame. First the ISS attitude estimate is transformed from ISS Navigation Coordinate Frame to the MUSES Platform Frame, then both ISS and MUSES attitude estimates are projected into the LVLH reference frame in terms of track-relative angles. The differences between MUSES and ISS track relative angles are defined herein as the Along-Track and Cross-Track Disparity, or Track Angle Disparities (TAD). Temporal and spectral characteristics of the resulting Track Angle Disparities form the basis for the initial MUSES attitude estimate analysis. To illustrate, Fig. 5 shows the ISS and PCS (MUSES EKF) Along-Track and Cross-Track Angle time histories for January 5, 2018 (Julian Day 2458124). Bias offsets representing static misalignments between the near-parallel ISS body fixed and MUSES platform frames have been removed.

Taking the difference between ISS and PCS track angles results in the track angle disparities shown in Fig. 6.

The resulting track angle disparities for the day shown have a peak to peak disparity of approximately 0.09 deg and 0.06 deg in along-track and cross-track respectively, with Root-Mean-Square (RMS) values of 0.024 deg (1.42 arc minutes) and 0.017 deg (1.0 arc minutes) in along-track and cross-track respectively.

B. Spectral Content of Track Angle Disparities

To assess the frequency domain characteristics of the attitude estimates, consider a continuous time function, $x(t)$, which can be expressed as a Fourier Series by a weighted sum of sinusoids with different frequencies and amplitudes. Individual weights correspond to the relative contribution of each sinusoid. In the limit, the Fourier series generalizes to the Fourier Transform (and its inverse) which maps a time domain signal to its frequency domain analog. The Fourier transform pair for a continuous time signal, $x(t)$, can be written as

$$X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt \quad (1)$$

and

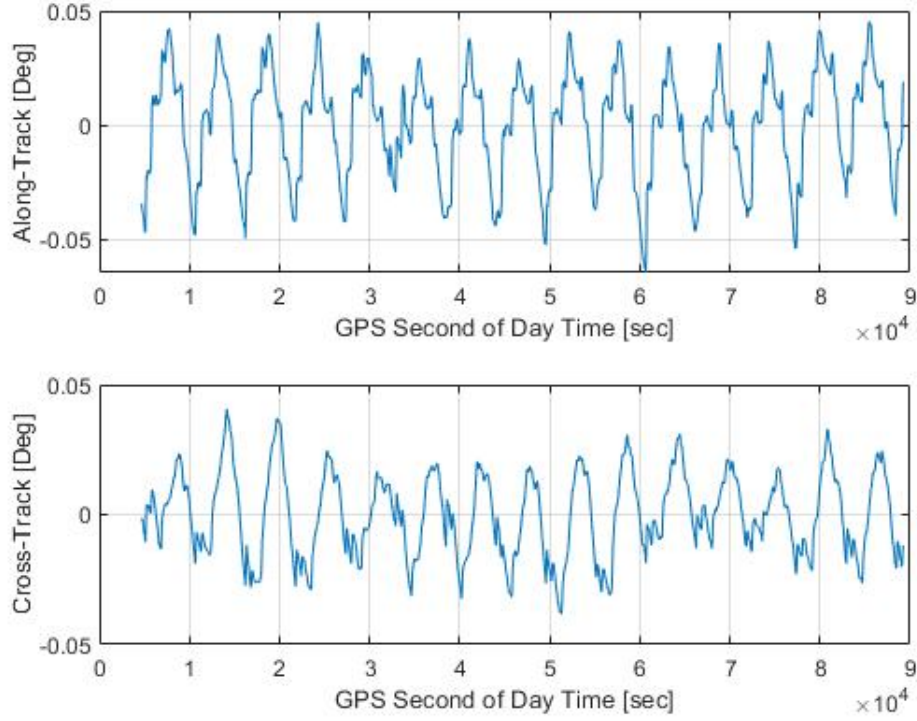


Fig. 6 Along- and Cross-Track Angle Disparities, January 5, 2018

$$x(t) = \int_{-\infty}^{\infty} X(\omega) e^{i\omega t} d\omega \quad (2)$$

For a sampled time series $\{x_t\}$, the Fourier Transform is

$$X(\omega) = \sum_{n=0}^{N-1} x_n e^{-i2\pi n k / N} \quad (3)$$

Assuming that the attitude error is a zero mean, normally distributed and stationary random process (e.g. the error is driven by random noise with biases removed), the autocovariance function of the time signal is defined as

$$\gamma_x(h) = E(x_t x_{t-h}) \quad (4)$$

Parseval's Theorem equates the total energy of a signal in the time domain and its frequency domain analog. Recognizing that variance is a measure of power in a time domain signal, and that the integral of power over time is energy allows us to employ Parseval's Theorem to define the Power Spectral Density (PSD) of the signal as the Fourier Transform of the autocovariance, or

$$S_x(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \gamma_x(h) e^{-ih\omega} dh \quad (5)$$

The Power Spectral Density describes how the power of a signal varies as a function of frequency and is often used to analyze the frequencies and magnitudes of noise in a measurement or vibratory modes in a structural dynamic system. Shown in Figs. 7 and 8 are the power spectral densities of the track-angle disparities for Day 5, 2018. Evident in these figures are the prominent peaks in power corresponding to integer multiples of the orbit frequency. These modes represent energy in the track angle disparity resulting from orbit-driven forcing functions that strongly contribute to the difference in the track angle estimates of the MUSES EKF and the ISS state vector. It is reasonable to conclude

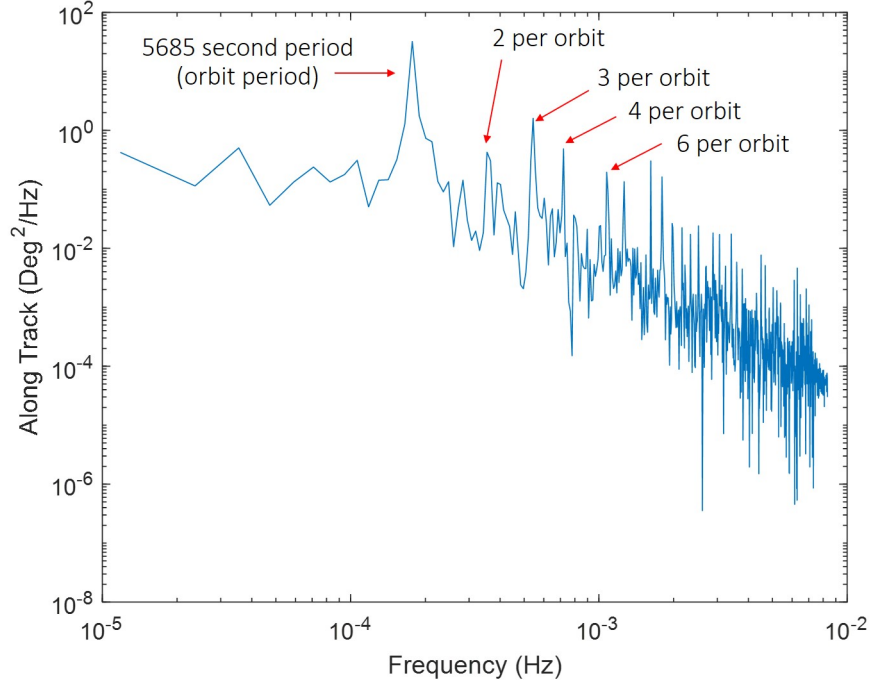


Fig. 7 Power Spectral Density of Along-Track Angle Disparity, January 5, 2018

that these orbit-correlated peaks in the PSD result from structural dynamics of the ISS, driven by atmospheric drag variations and variations in thermal loading as a function of shadowing and solar beta angle, and producing time-varying misalignments between the ISS body fixed frame and the MUSES platform frame. A plot of the power spectral densities in linear axes better indicates the relative dominance of the orbit driven modes as compared to the remaining spectral content of the track-angle disparities. Figures 9 and 10 show the relative contribution of the modes corresponding to the first six integer multiples of orbit frequency. Note that the contribution of the five cycles per orbit mode is considerably reduced in along-track as compared to the cross-track response.

These plots of the power spectral density indicate a strong contribution of orbit-correlated energy in the track angle disparities and suggest that a more accurate, “Residual Attitude Error” estimate can be obtained by removing the contribution of energy in the track-angle disparities that corresponds to orbit-driven flexural dynamics of the ISS. The following section addresses the methodology for removing spectral components from a time domain signal to obtain an accurate Residual Attitude Error estimate and presents results for the MUSES EKF attitude estimate accuracy.

C. Orbit Driven Misalignment Variance

Parseval’s theorem equates the energy of a signal in the time and frequency domains and shows that the total energy in a time signal is the result of the combined energies from all spectral components of the signal. As a consequence, the variance in the time domain signal is equivalent to the PSD integrated over all frequencies, or equivalently, to the sum of the integrated PSD over discrete, constituent frequency bands. Parseval’s Theorem thus enables the PSD to be disaggregated into the spectral components of the signal variance.

Given the PSD, $S_x(\omega)$, the autocovariance is obtained using the inverse Fourier Transform

$$\gamma_x(h) = \int_{-\infty}^{\infty} S_x(\omega) e^{i\omega h} d\omega \quad (6)$$

Evaluating the autocovariance at $h = 0$ yields the variance of $\{x_t\}$, or the mean-square value, $\gamma_x(0) = E(x_t^2) = \sigma^2$. The variance of the time series is then obtained by integrating the PSD over frequency as

$$\sigma^2 = \int_{-\infty}^{\infty} S_x(\omega) d\omega \quad (7)$$

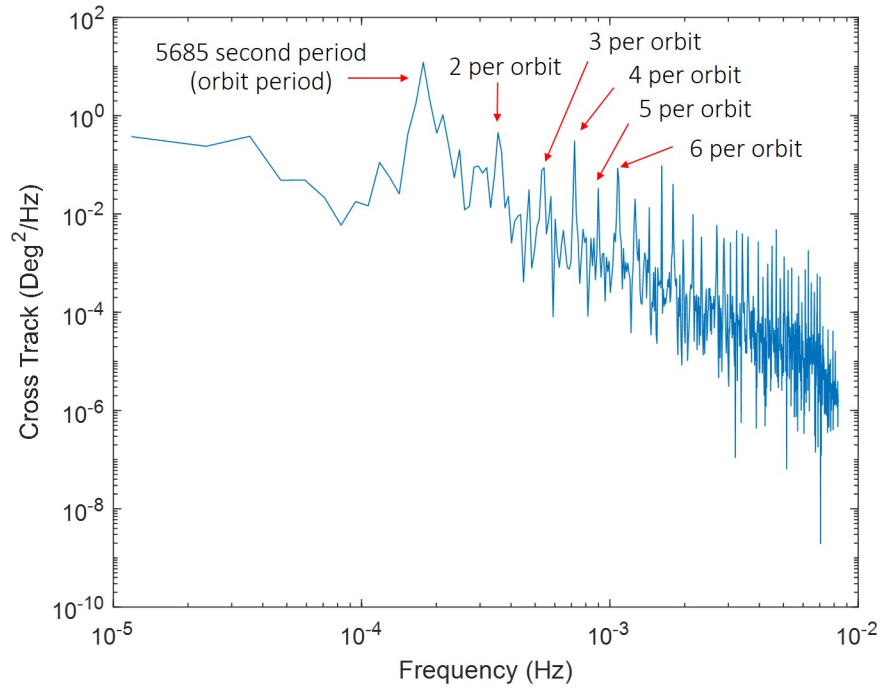


Fig. 8 Power Spectral Density of Cross-Track Angle Disparity, January 5, 2018

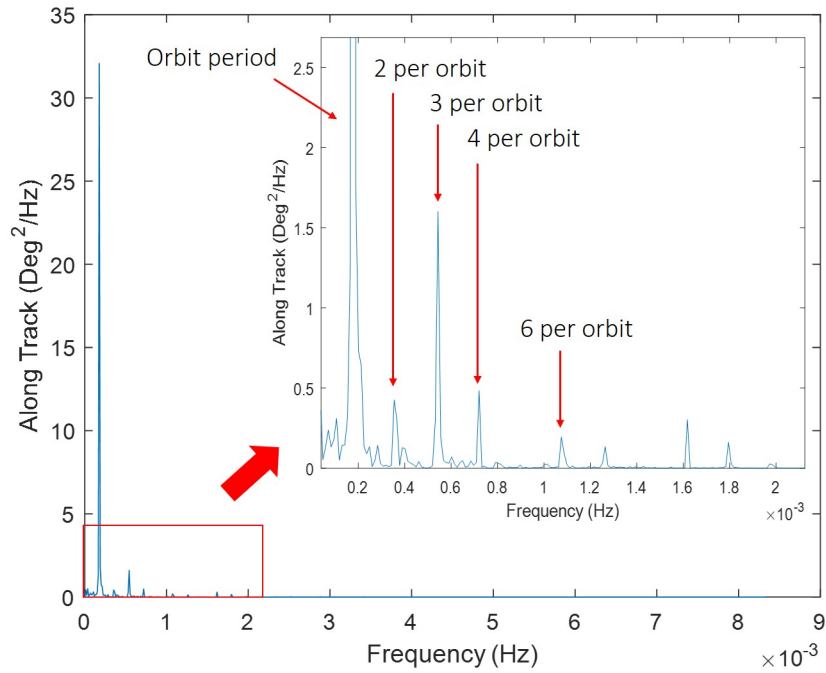


Fig. 9 Linear Along-Track Angle Disparity PSD, January 5, 2018

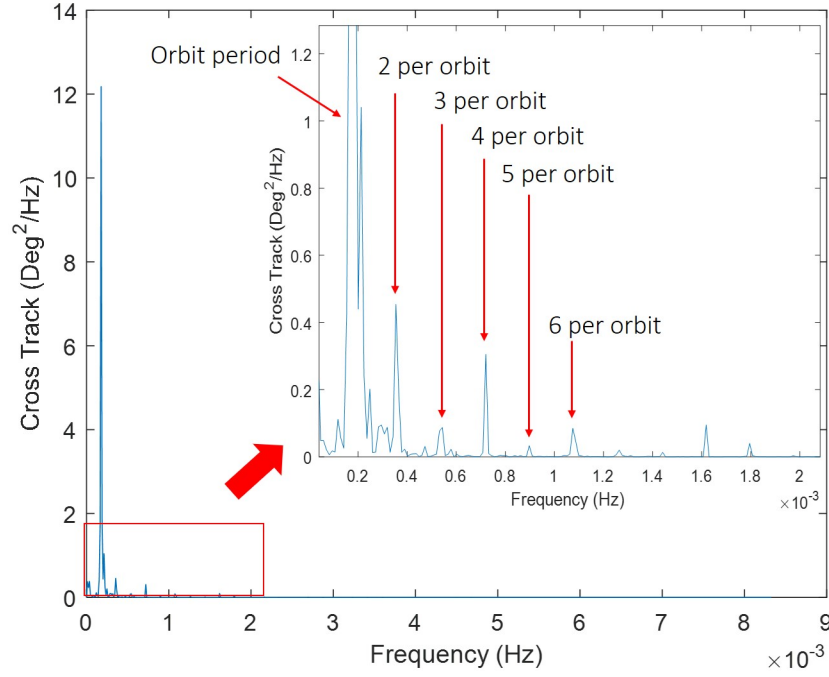


Fig. 10 Linear Cross-Track Angle Disparity PSD, January 5, 2018

Summing the numerically integrated PSD from each sampling interval over the entire frequency series yields a cumulative mean square that is approximately equivalent to the variance of the time sample. For example, the variance of the Along-Track Angle Disparity time series for January 5, 2018 is $5.4982\text{e-}04 \text{ deg}^2$, while summing the integrated PSD over each distinct frequency band leads to a cumulative mean square value of $5.4943\text{e-}04 \text{ deg}^2$.

By defining limits of integration as finite frequency intervals, we can calculate the time-domain variance, or mean-square, constituent value as a function of individual frequency bands. The constituent variance, obtained by integrating the PSD across an individual frequency band from f_1 to f_2 , is shown graphically in Fig. 11.

The number of PSD data points in the frequency series as well as the resolution of the frequency sample points is determined by the length of the time sample and the time sampling interval. The frequency resolution, Δf , or spacing between consecutive data points in Hz is $1/T$, where T is the total span in seconds of the time series. Thus a longer duration time series gives a finer resolution of the frequency series. For January 5, 2018 track angle discrepancies, the entire time series is comprised of 1,411 data points sampled once per minute, corresponding to approximately 15.5 orbits. The resulting Constituent RMS obtained from taking the square root of the integrated Power Spectral Density

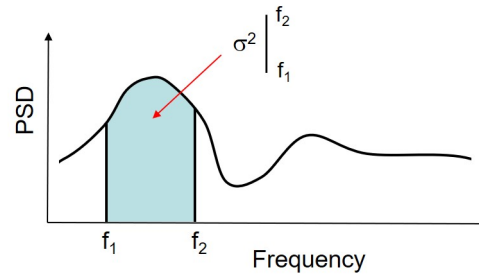


Fig. 11 Definition of Constituent Mean-Square from Integrated Power Spectral Density

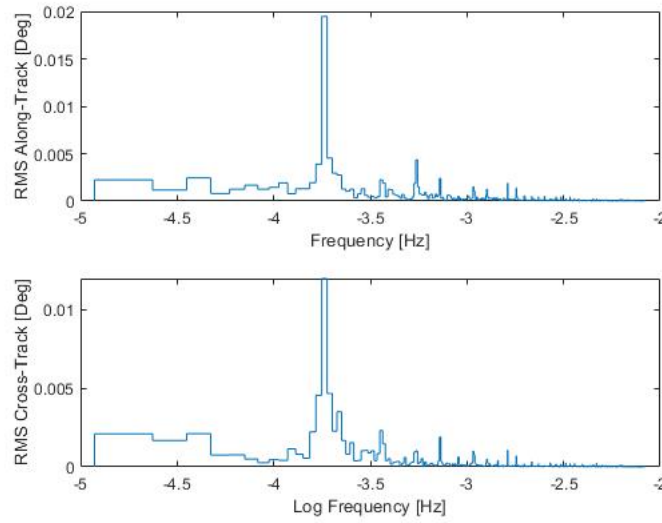


Fig. 12 Constituent RMS from Integrated PSD of TAD, Day 5, 2018

of the Along- and Cross-Track Disparities is shown in Fig. 12. Note the effective spectral decomposition of the total variance that is implied in Fig. 12, where the relative magnitude of individual peaks in the integrated PSD indicate the relative contribution of each spectral band to the total variance.

IV. Long-Term MUSES Attitude Estimation Analysis Results

The power spectra of the track angle disparities can be assessed over multiple days to show the variation in magnitude of the orbit-driven response modes. The following analysis is conducted for 280 "quiescent" days from January 1, 2018 through November 23, 2019. For the purposes of this analysis, a quiescent day relates to an extended time of multiple orbits on a given day during which Track Angle Disparities are attributable to MUSES or ISS attitude error. For example, MUSES Slews result in a large Track Angle Disparity and must be excluded from the calculation of attitude error. When a sufficient number of orbits remain on a given day, the portion of the time histories containing the slew maneuvers is truncated from the analysis data. Figure 13 illustrates the approach to truncation to retain multiple orbits to comprise a quiescent day time record for the Track Angle Disparity analysis. After slew orbits are truncated from the data set, the remaining quiescent time history for the analysis day is shown in Fig. 14. This segment of data more correctly represents the statistical performance of the MUSES attitude estimate in the calculation of Track Angle Disparity.

ISS attitude maneuvers also introduce significant track angle disparities as a result of transient loads and increased excitation of structural modes and as such do not represent time periods where the Track Angle Disparity is due to ISS or MUSES attitude error. Portions of analysis days with ISS maneuvers are truncated with the remaining orbits comprising the quiescent portion of that analysis day as described above.

Figures 15 and 16 show the constituent variance contributions of the orbit-driven response modes over this extended analysis period over Julian Days 2458113 – 2458338. Each tick mark is the per-day MS value obtained from the integrated PSD for each orbit-correlated frequency. The variation in magnitude at each orbit-correlated frequency indicates how the relative alignment between the MUSES Platform Frame and ISS Nav Frame varies from day-to-day based on changes in the orbit environment.

Figures 15 and 16 clearly indicate a strong qualitative correlation between track-angle disparities and integer multiples of orbit period as well as the varying energy contributed over multiple days. Future work on modeling the orbit-driven ISS deformations in the EKF can potentially account for these track-angle disparities and enable the MUSES attitude to be directly estimated from the ISS state in the event that the MUSES SED26 star tracker degrades or fails. Figures 17 and 18 show the contribution of the first 6 orbit-correlated modes to the total variance in the Track Angle Disparity over the same extended analysis period (plotted against calendar days beginning January 1, 2018). Clearly the primary constituent of Track Angle Disparities is the mode (shown in blue) correlated to orbit period which can be presumed a result of ISS deformations driven at the orbit period. The remaining modes contribute approximately 10

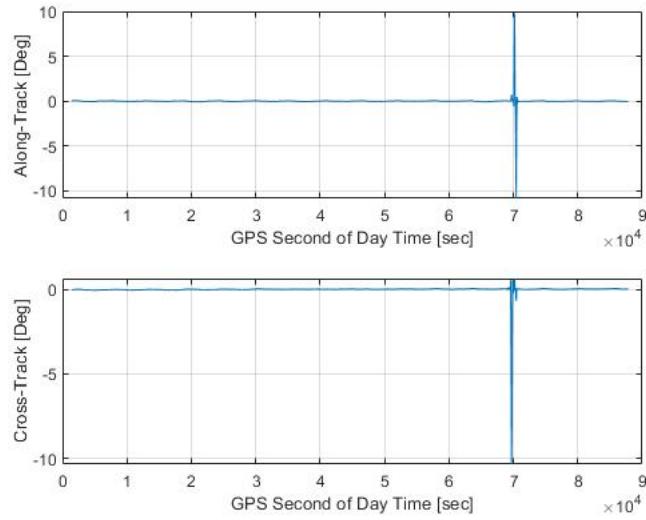


Fig. 13 MUSES Slew Induced Track Angle Disparity

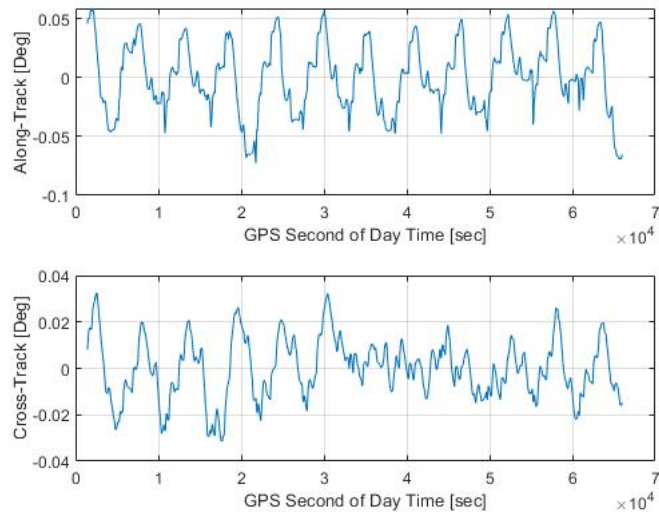


Fig. 14 Retained Quiescent Time Segment

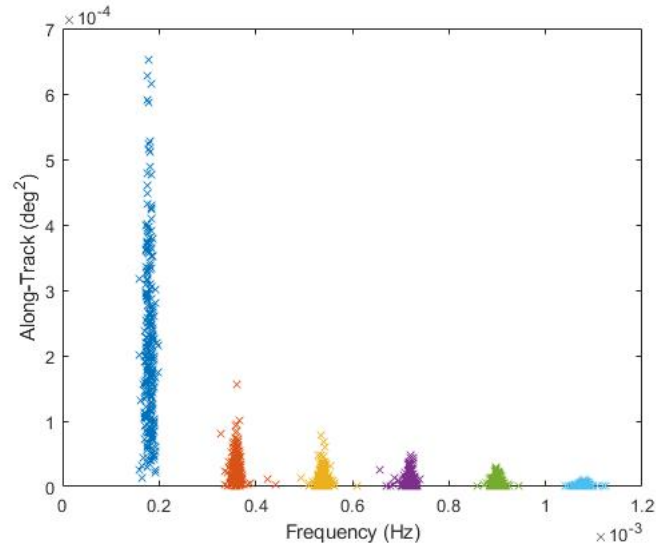


Fig. 15 Constituent Variance Contributions to Along-Track Angle Disparities

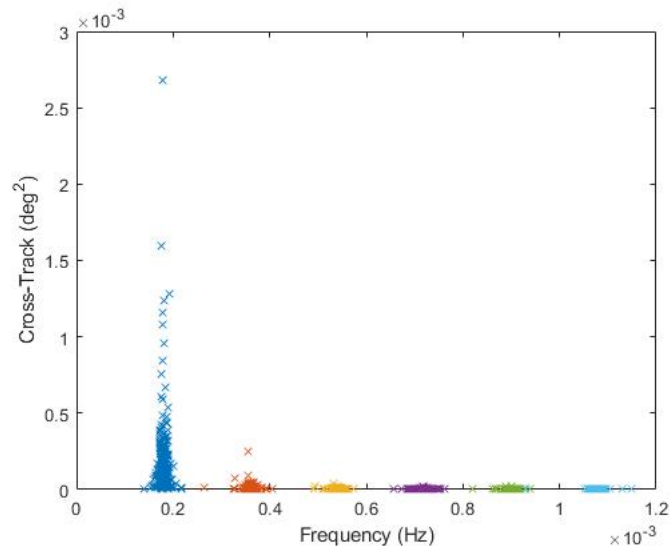


Fig. 16 Constituent Variance Contributions to Cross-Track Angle Disparities

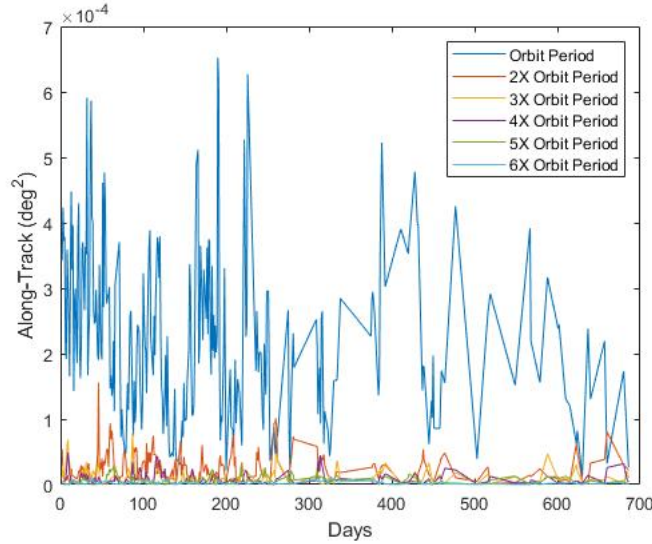


Fig. 17 Along-Track Orbit-Correlated Constituent Mode Contribution to Cumulative Mean Square

percent or less to the total variance.

A. Residual Attitude Error Estimation

The Residual Attitude Error estimate is obtained by removing each orbit-correlated mode contribution from the cumulative mean square value of the track angle disparity. The contribution of each orbit-correlated mode is obtained from integrating the PSD over the sample frequency band corresponding to the orbit-correlated frequency. For this analysis the first 6 orbit-correlated modes are estimated and removed. The resulting per-day Residual Attitude Errors are plotted in Figs. 19 and 20 for Along-Track and Cross-Track, respectively, with the solid lines indicating the average values over the 280 quiescent days of this long term analysis period.

Finally, note that both ISS attitude error and MUSES PCS attitude error combine in the estimated Residual Attitude Error defined above. Both MUSES and ISS attitude estimates are independently generated, optimal least squares error estimates and hence, the Residual Attitude Error can be presumed to be the vector sum of two uncorrelated error signals. The MUSES Attitude Error Estimate is then obtained by dividing the Residual Attitude Error variance in half and taking the square root. The result of this process – forming the Track Angle Disparities, calculating the power spectral density, disaggregating the spectral components, removing the orbit-driven ISS deformations, and extracting the MUSES component of the resulting Residual Attitude Error – is the independent “best estimate” of MUSES attitude error. For the 280 quiescent analysis days during January 1, 2018 - November 23, 2019, the average MUSES attitude error is 34.5 and 27.7 arc seconds in Along-Track and Cross-Track respectively. Considering that the minimum single-day attitude errors constitute the best estimate, or least upper bound, of MUSES attitude errors, the long term best estimate was obtained for July 19, 2018 as follows:

Best Estimate of MUSES Attitude Errors:
Along-Track MUSES Attitude Error = 12.0 arc seconds
Cross-Track MUSES Attitude Error = 9.47 arc seconds

These analyses demonstrate the value of the Track Angle Disparity as a metric for assessing the performance of the MUSES PCS for estimating the inertial pointing orientation of the instrument platform. Additional analyses over extended periods of time will result in a more accurate, long term average as well as potentially indicate temporal trends that can be modeled and removed in EKF refinements. More importantly, monitoring the daily values of MUSES Residual Attitude Error will be an important operational metric to ensure continued stable and consistent operations of the MUSES platform and Pointing Control System.

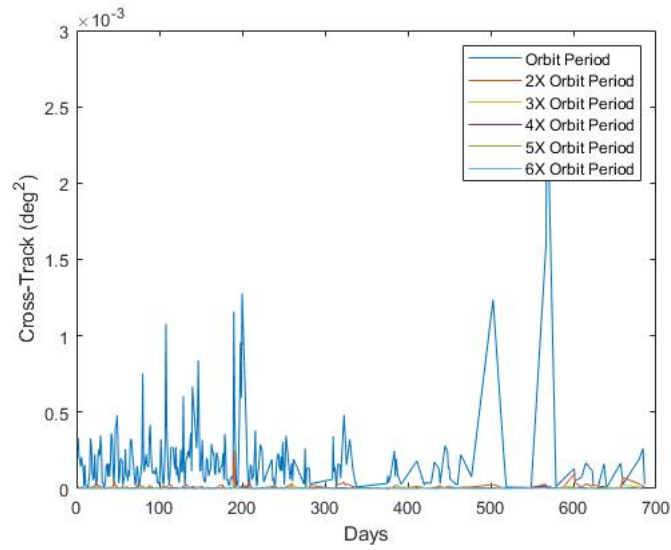


Fig. 18 Cross-Track Orbit-Correlated Constituent Mode Contribution to Cumulative Mean Square

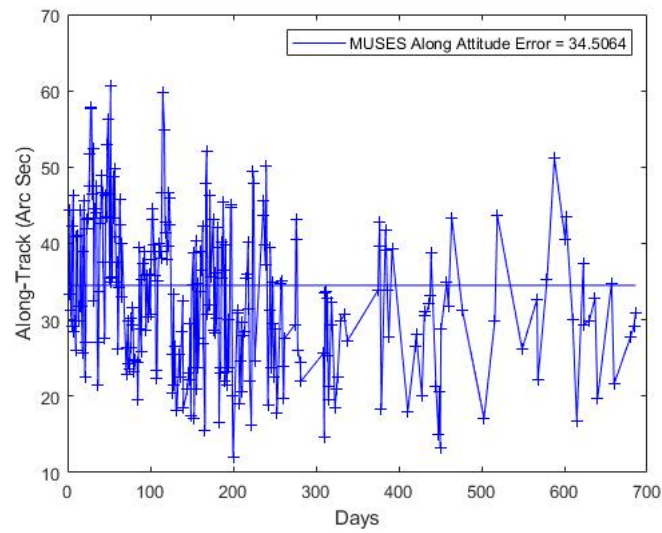


Fig. 19 Along-Track Cumulative and Residual Attitude Error values

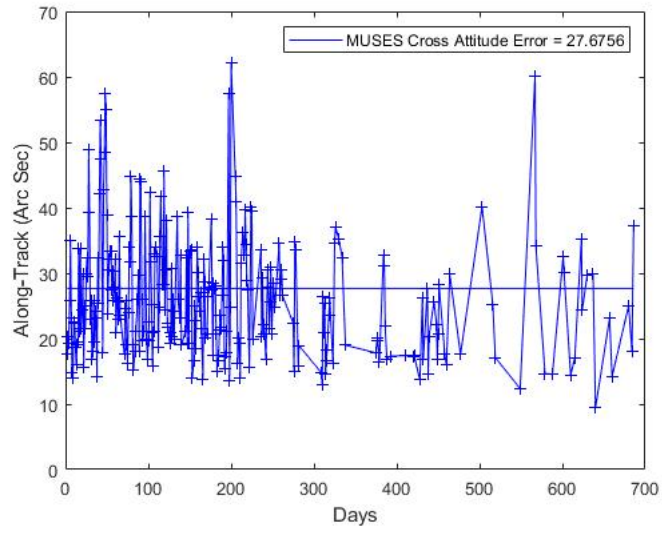


Fig. 20 Cross-Track Cumulative and Residual Attitude Error values

V. References

1. MUSES PCS Software System Description Document, July 2013.
2. Crassidis, J. L., and Whorton, M. S., "Multi-User System for Earth Sensing Spacecraft Attitude Estimation Analysis", submitted to the 2020 AIAA Scitech Conference.